1-5 Variables and Equations

- Identify and solve open sentences.
- Translate verbal sentences into equations.

Vocabulary

- equation
- open sentence
- solution
- solving the equation

EQUATIONS AND OPEN SENTENCES

A mathematical sentence that contains an equals sign (=) is called an **equation**. A few examples are shown.
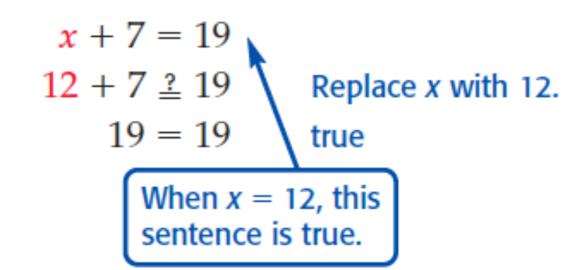
$$5 + 9 = 14 \qquad 2(6) - 3 = 9 \qquad x + 7 = 19 \qquad 2m - 1 = 13$$

An equation that contains a variable is an **open sentence**. An open sentence is neither true nor false. When the variable in an open sentence is replaced with a number, you can determine whether the sentence is true or false.

$x + 7 = 19$

$11 + 7 \stackrel{?}{=} 19$ Replace x with 11.

$18 \neq 19$ false

> When $x = 11$, this sentence is false.

$x + 7 = 19$

$12 + 7 \stackrel{?}{=} 19$ Replace x with 12.

$19 = 19$ true

> When $x = 12$, this sentence is true.

Example **1** **Solve an Equation**

Find the solution of $12 - m = 8$. Is it 2, 4, or 7?

Replace m with each value.

Value for m	$12 - m = 8$	True or False?
2	$12 - 2 \stackrel{?}{=} 8$	false
4	$12 - 4 \stackrel{?}{=} 8$	true ✓
7	$12 - 7 \stackrel{?}{=} 8$	false

Therefore, the solution of $12 - m = 8$ is 4.

Example 2 *Solve an Equation*

Multiple-Choice Test Item

Which value is the solution of $2x + 1 = 7$?

  (A) 6   (B) 5   (C) 4   (D) 3

Read the Test Item

The *solution* is the value that makes the equation true.

Solve the Test Item Test each value.

$$2x + 1 = 7$$
$$2(6) + 1 = 7 \quad \text{Replace } x \text{ with 6.}$$
$$13 \neq 7$$

$$2x + 1 = 7$$
$$2(5) + 1 = 7 \quad \text{Replace } x \text{ with 5.}$$
$$11 \neq 7$$

$$2x + 1 = 7$$
$$2(4) + 1 = 7 \quad \text{Replace } x \text{ with 4.}$$
$$9 \neq 7$$

$$2x + 1 = 7$$
$$2(3) + 1 = 7 \quad \text{Replace } x \text{ with 3.}$$
$$7 = 7 \ \checkmark$$

Since 3 makes the equation true, the answer is D.

Example 3 **Solve Simple Equations Mentally**

Solve each equation mentally.

a. $5x = 30$

$5 \cdot 6 = 30$ Think: What number times 5 is 30?

$x = 6$ The solution is 6.

b. $\dfrac{72}{d} = 8$

$\dfrac{72}{9} = 8$ Think: 72 divided by what number is 8?

$d = 9$ The solution is 9.

Reading Math

Symmetric

Root Word: Symmetry

The word *symmetry* means *similarity of form or arrangement on either side.*

Key Concept — Properties of Equality

Property	Words	Symbols	Example
Symmetric	If one quantity equals a second quantity, then the second quantity also equals the first.	For any numbers a and b, if $a = b$, then $b = a$.	If $10 = 4 + 6$, then $4 + 6 = 10$.
Transitive	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first equals the third.	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $3 + 5 = 8$ and $8 = 2(4)$, then $3 + 5 = 2(4)$.

Example 4 **Identify Properties of Equality**

Name the property of equality shown by each statement.

a. If $5 = x + 2$, then $x + 2 = 5$.

If $a = b$, then $b = a$. This is the Symmetric Property of Equality.

b. If $y + 8 = 15$ and $15 = 7 + 8$, then $y + 8 = 7 + 8$.

If $a = b$ and $b = c$, then $a = c$. This is the Transitive Property of Equality.

Example **5** **Translate Sentences Into Equations**

The difference of a number and ten is seventeen. Find the number.

Words   The difference of a number and ten is seventeen.

Variables   Let $n =$ the number.   Define the variable.

$$\underbrace{\text{The difference of a number and ten}}_{} \qquad \underbrace{\text{is}}_{} \qquad \underbrace{\text{seventeen.}}_{}$$

Equation $n - 10$ $=$ 17

$n - 10 = 17$ Write the equation.

$27 - 10 = 17$ Think: What number minus 10 is 17?

$n = 27$ The solution is 27.